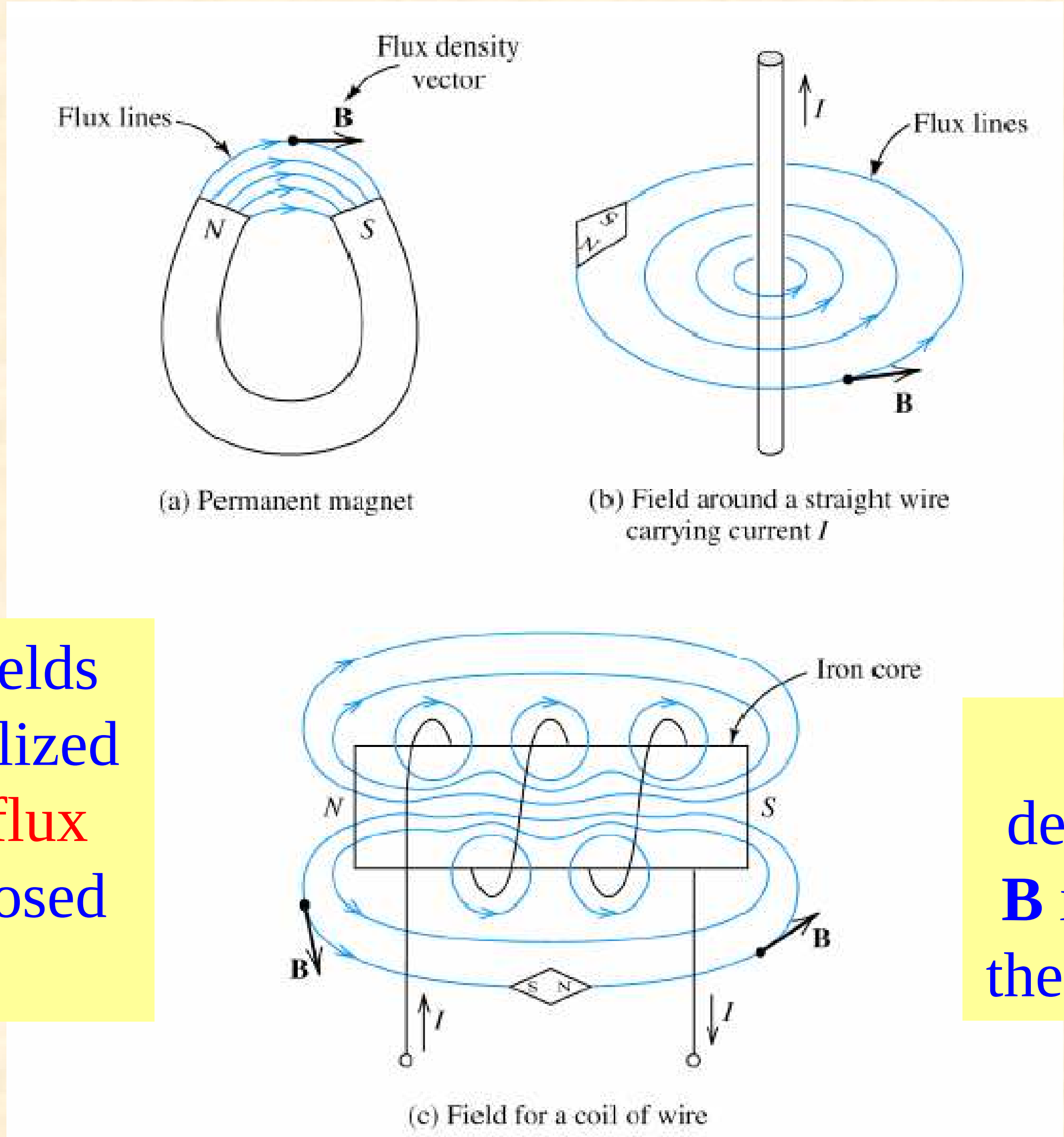
Part 3

Mutual Inductance

Main Outlines

- Review of self inductance
- Concept of mutual inductance
- Mutual inductance in terms of self inductance
- Polarity of the mutually induced voltages (Dot Convention)
- Procedure for determining dot marking
- Use of dot markings in circuit analysis
- Energy calculations

Magnetic Field



Magnetic fields can be visualized as lines of flux that form closed paths

The flux density vector **B** is tangent to the lines of flux

Magnetic Field

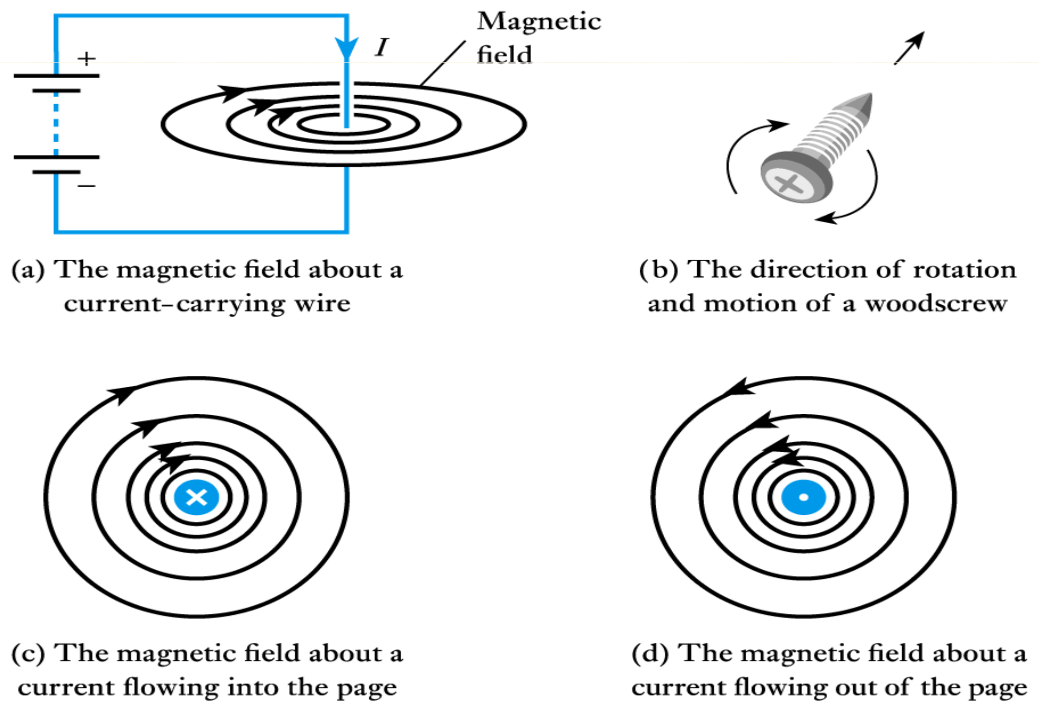
- Magnetic flux lines form closed paths that are close together where the field is strong and farther apart where the field is weak

- Flux lines leave the north end (pole) of a magnet and enter the south end (pole)

- When placed in a magnetic field, a compass indicates north in the direction of the flux lines

Magnetic Field

- A wire carrying a current *I* causes a **magneto-motive force (m.m.f) *F***
  - this produces a **magnetic field**
  - ***F*** has units of Amperes
  - for a single wire *F* is equal to *I*



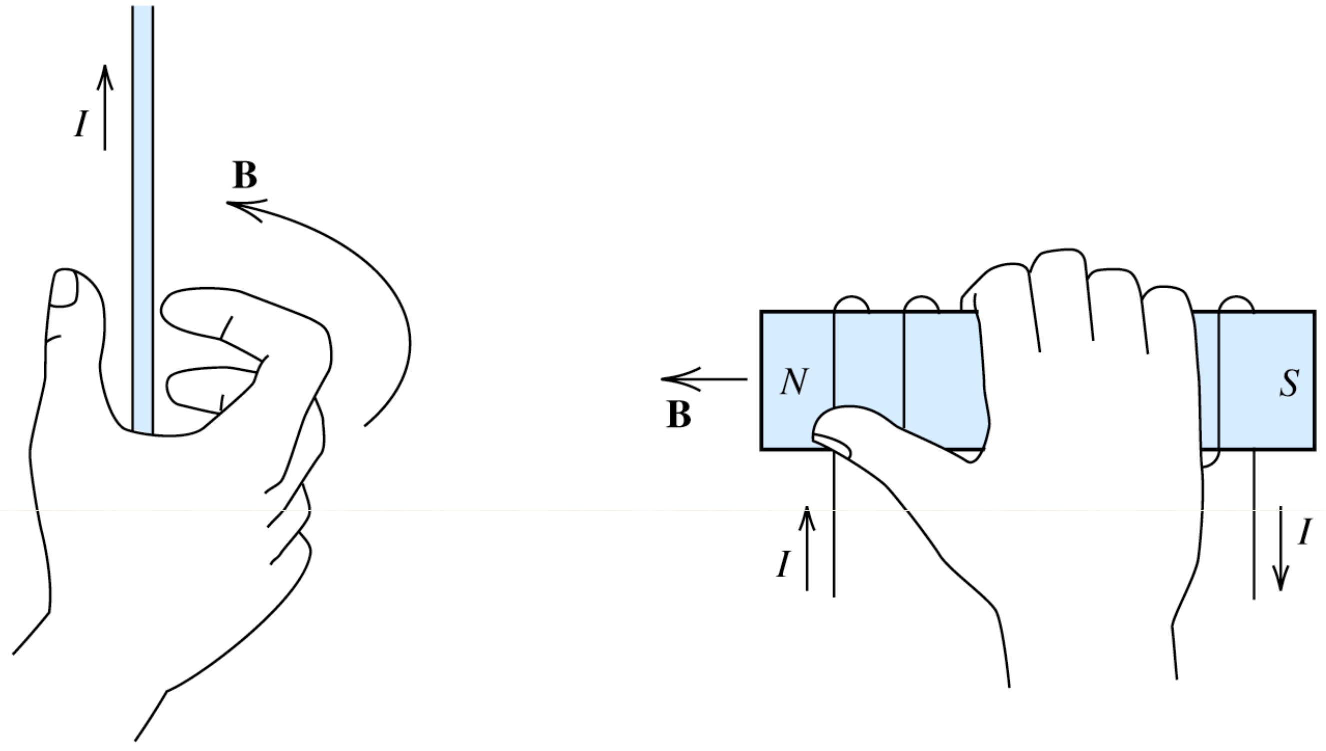
(a) The magnetic field about a current-carrying wire

(b) The direction of rotation and motion of a woodscrew

(c) The magnetic field about a current flowing into the page

(d) The magnetic field about a current flowing out of the page

Magnetic Field



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Right-Hand Rule

Magnetic Field

- The magnitude of the field is defined by the **magnetic field strength (intensity)**, $\boldsymbol{H}$, where

$$H = \frac{I}{l}$$

where l is the length of the magnetic circuit

- The magnetic field produces a **magnetic flux**, $\boldsymbol{\Phi}$
  - flux has units of weber (Wb)

- Strength of the flux at a particular location is measured in term of the **magnetic flux density**, $\boldsymbol{B}$
  - flux density has units of tesla (T) (equivalent to 1 Wb/m^2)

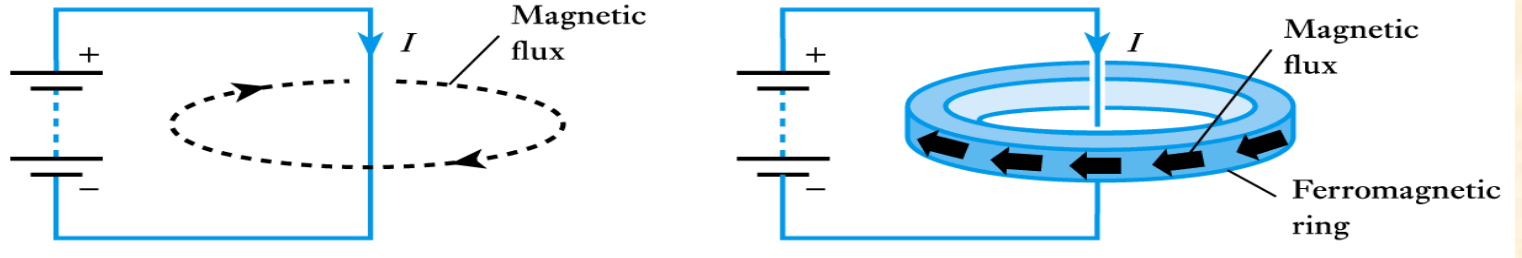
Magnetic Field

- Flux density at a point is determined by the field strength and the material present

$$B = \mu H \quad \text{or} \quad B = \mu_0 \mu_r H$$

- where μ is the permeability of the material, μ_r is the relative permeability and μ_0 is the permeability of free space

- Adding a ferromagnetic ring around a wire will increase the flux by several orders of magnitude, since μ_r for ferromagnetic materials is 1000 or more



(a) The magnetic flux about a current-carrying wire in air

(b) The effect of adding a ferromagnetic ring

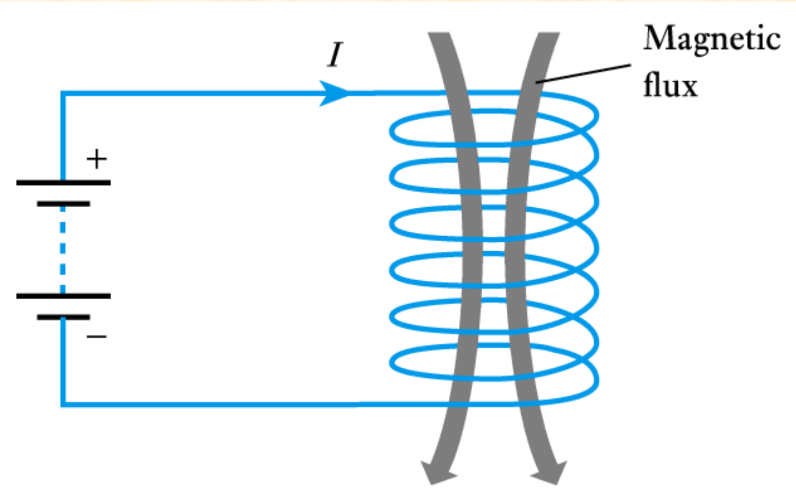
Magnetic Field

- When a current-carrying wire is formed into a **coil,** the magnetic field is **concentrated**

- For a coil of N turns the m.m.f. (F) is given by:

$$F = IN$$

and the field strength is $$H = \frac{IN}{l}$$



Magnetic Reluctance

- In a *resistive* circuit, the resistance is a measure of how the circuit opposes the flow of electricity
- In a *magnetic* circuit, the **reluctance**, $\Re$ is a measure of how the circuit opposes the flow of magnetic flux

✓ In a resistive circuit $R = V/I$

✓ In a magnetic circuit $\Re = \dfrac{F}{\Phi}$

- the units of reluctance are amperes per weber (A/ Wb)
- The magnetic **Permeance** is given by: $P = \dfrac{1}{\Re}$

Flux Linkages and Faraday's Law

- Magnetic flux passing through a surface area A:

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

- For a constant magnetic flux density perpendicular to the surface: $\phi = B\,A$

- The flux linking a coil with N turns:

$$\lambda = N\,\phi$$

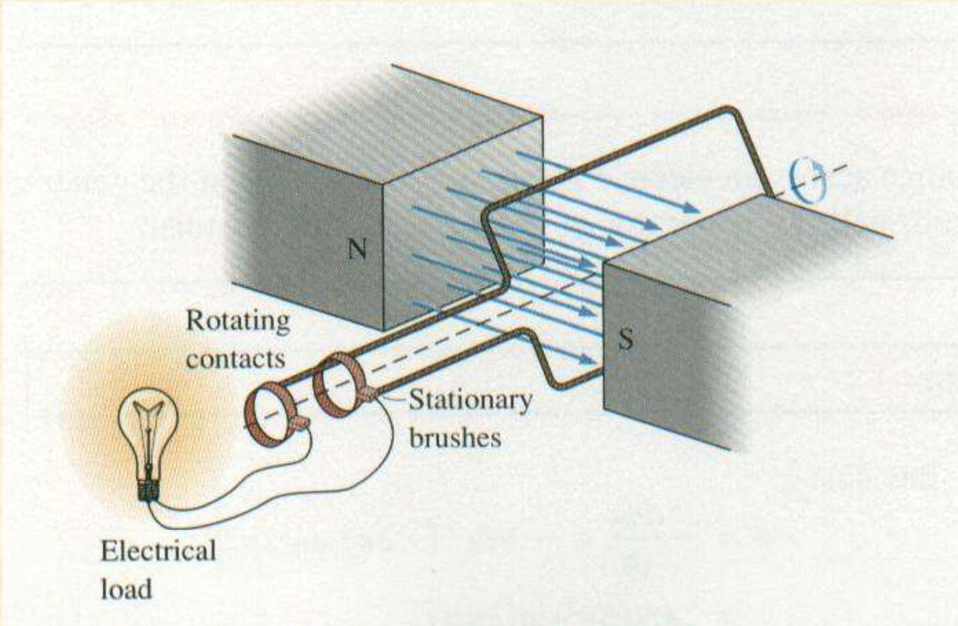
Faraday's Law

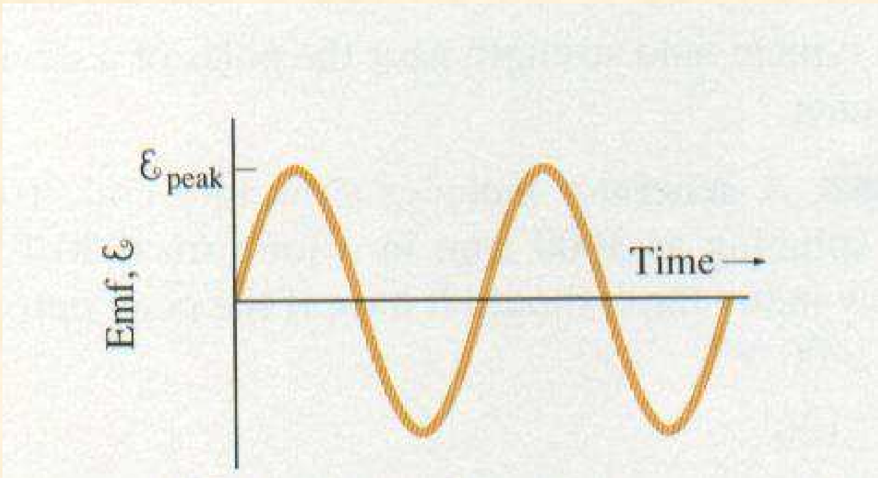
- **Faraday's law of magnetic induction:**

$$e = \frac{d\lambda}{dt}$$

- The voltage induced in a coil whenever its flux linkages are changing.
- Changes occur from:
  - Magnetic field changing in time
  - Coil moving relative to magnetic field

Faraday’s Law



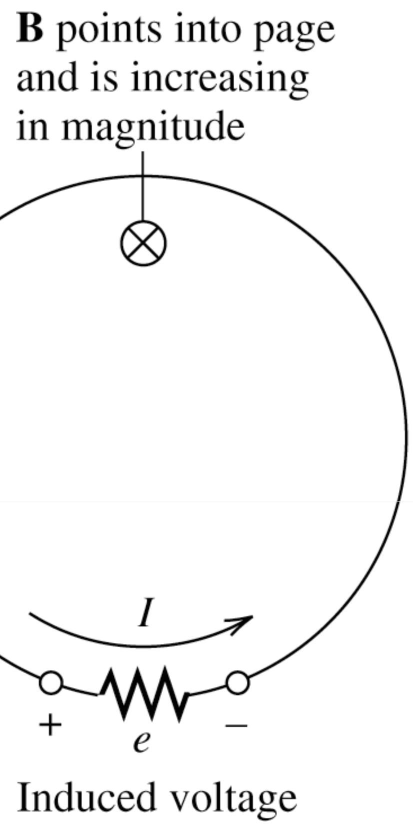


Lenz's Law

- **Lenz's law** states that the polarity of the induced voltage is such that the voltage would <u>produce a current</u> (through an external resistance) that opposes the original change in flux linkages.

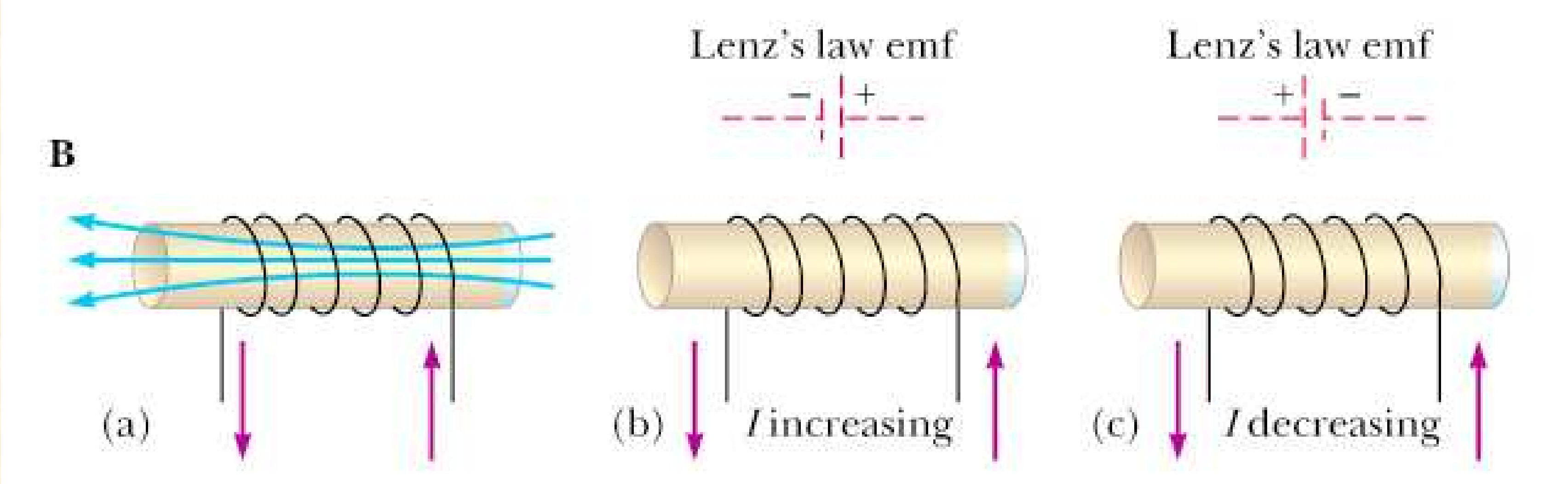
- Induced voltage at every instant opposes any change in circuit current
- The current in a conductor, as a result of an induced voltage, is such that the change in magnetic flux due to it is opposite to the change in flux that caused the induced voltage

Lenz's Law



When the flux linking a coil changes, a voltage is induced in the coil. The polarity of the voltage is such that if a circuit is formed by placing a resistance across the coil terminals, the resulting current produces a field that tends to oppose the original change in the field.

Lenz's Law



➢ When *I* changes, an emf is induced in the coil

➢ If *I* is increasing (and therefore increasing the flux through the coil), then the induced emf will set up a magnetic field to oppose the increase in the magnetic flux in the direction shown.

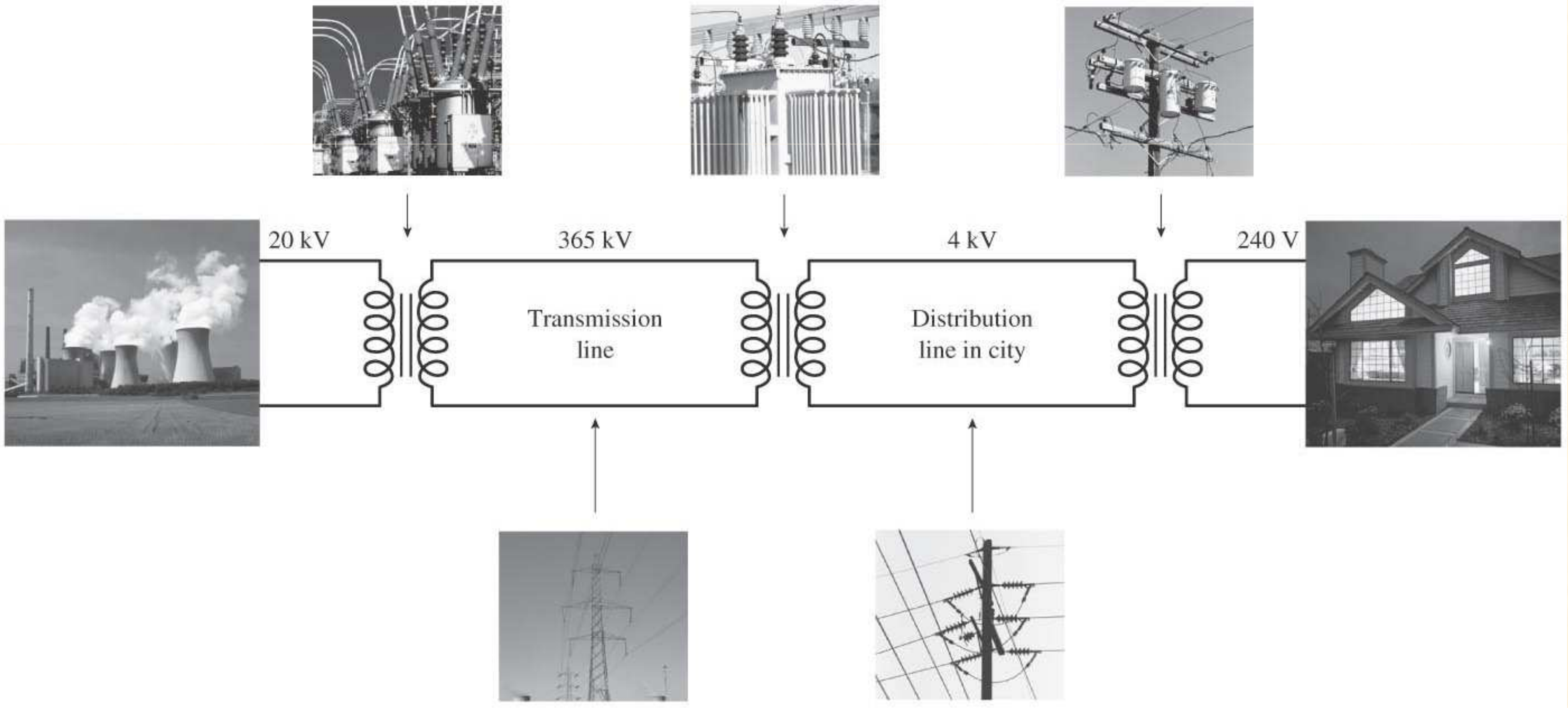
➢ If *I* is decreasing, then the induced emf will set up a magnetic field to oppose the decrease in the magnetic flux.

Magnetically Coupled Circuits

- When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, it called ***magnetically coupled***
- **Example: transformer**
  - ✓ An electrical device designed on the basis of the concept of magnetic coupling
  - ✓ Used magnetically coupled coils to transfer energy from one circuit to another

Transformers and Power Transmission

- Electric power is most efficiently transmitted at high voltages.
  - This reduces I^2R energy losses in the power lines.
  - But most end uses require lower voltages.
  - Transformers accomplish voltage changes throughout the power grid.

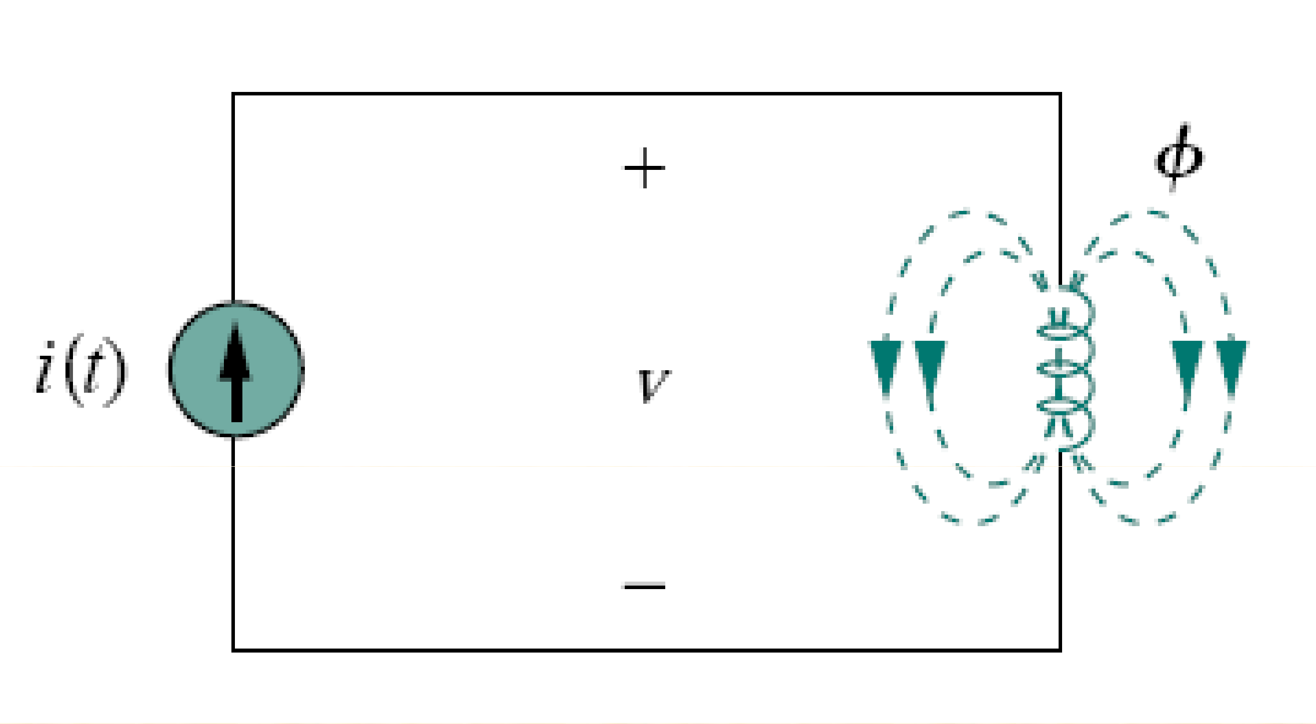


Self and Mutual Inductance

- 1 coil (inductor)
  - Single solenoid has only self-inductance (L)

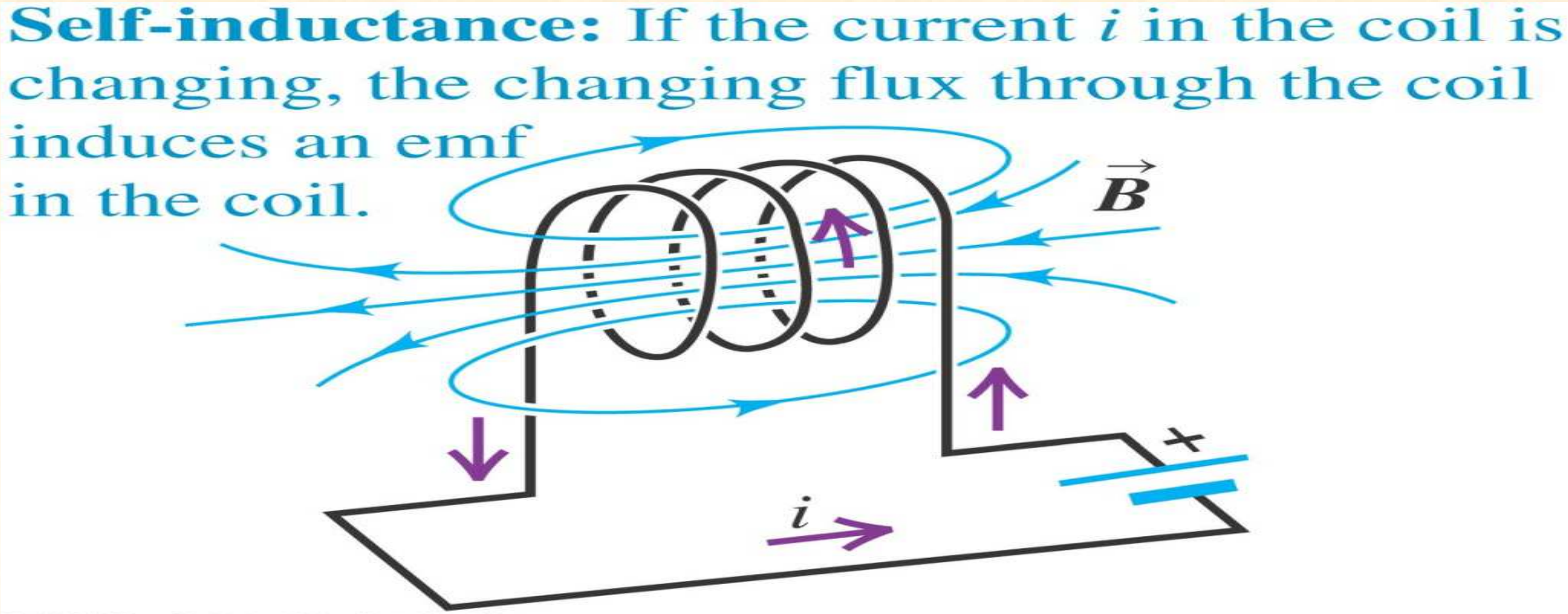
- 2 coils (inductors)
  - 2 solenoids have self-inductance (L) & Mutual-inductance

Self Inductance



- ✓ A coil with N turns produced ϕ = magnetic flux
- ✓ Only has self inductance, L

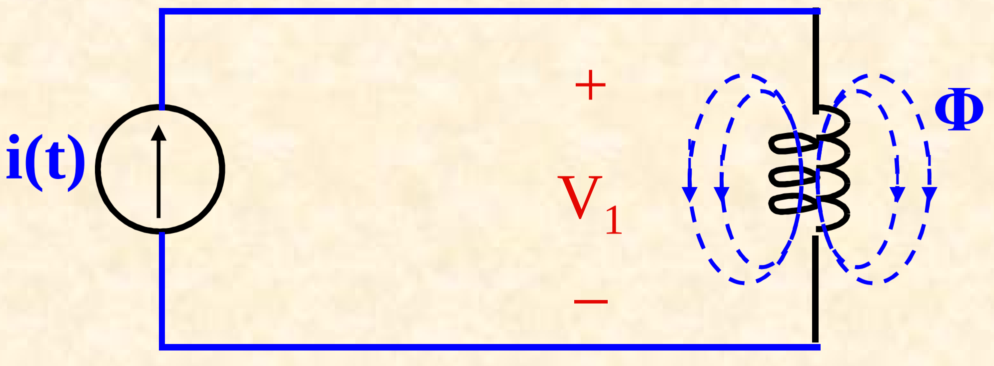
Self Inductance



The current *i* in the circuit causes a magnetic field $\underline{B}$ in the coil and hence a flux through the coil. When the current **changes**, the flux changes also and a self-induced emf appears.

Self Inductance

- It called *self inductance* because it relates the voltage induced in a coil by a time varying current in the same coil
- Consider a single inductor with N number of turns when current, *i* flows through the coil, a magnetic flux, Φ is produces around it



Self Inductance

- According to Faraday's Law, the voltage, *v* induced in the coil is proportional to N number of turns and rate of change of the magnetic flux, Φ;

$$v = N\frac{d\phi}{dt} \text{.......(1)}$$

- But a change in the flux Φ is caused by a change in current, *i.* Hence;

$$\frac{d\phi}{dt} = \frac{d\phi}{di}\frac{di}{dt} \text{.......(2)}$$

Self Inductance

- Thus, (2) into (1) yields;

$$v = N\frac{d\phi}{di}\frac{di}{dt}.......(3)$$

or

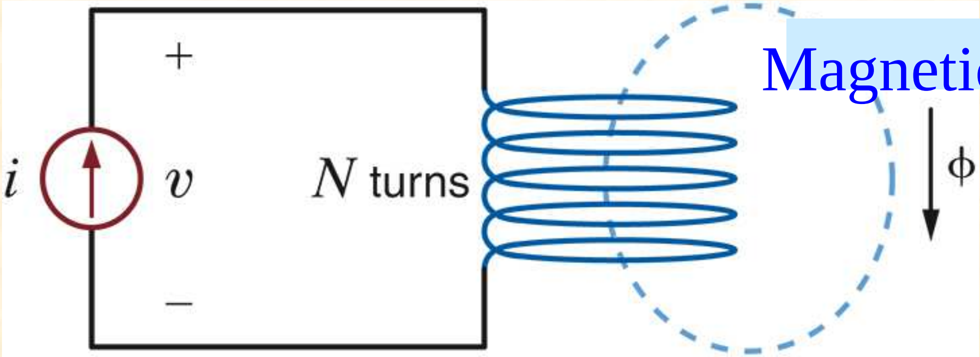
$$v = L\frac{di}{dt}..............(4)$$

- From equation (3) and (4) the self inductance L is define as;

$$L = N\frac{d\phi}{di} \quad (\mathrm{H})........(5)$$

✓ The unit is in Henry (H)

Self Inductance (conclusions)



Magnetic field

$$\lambda = N\phi$$

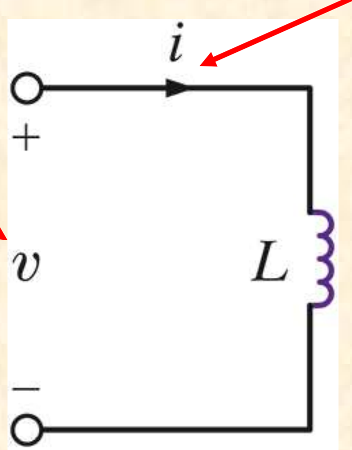
Total magnetic flux linked by N-turn coil

$$v = \frac{d\lambda}{dt}$$

Faraday's Induction Law

$$\lambda = Li$$

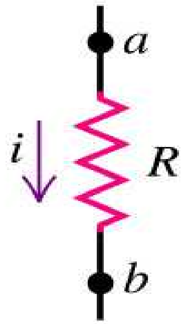
Ampere's Law



$$v = L\frac{di}{dt}$$

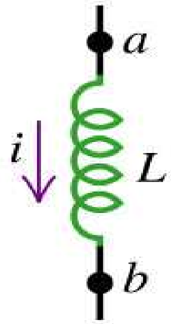
Ideal Inductor

Resistor and Inductor



$$V_{ab} = iR$$

(a) Resistor with current i flowing from a to b: potential drops from a to b



$$V_{ab} = L\frac{di}{dt}$$

(b) Inductor with current i flowing from a to b:
- If $di/dt > 0$: potential drops from a to b
- If $di/dt < 0$: potential increases from a to b
- If i is constant ($di/dt = 0$): no potential difference

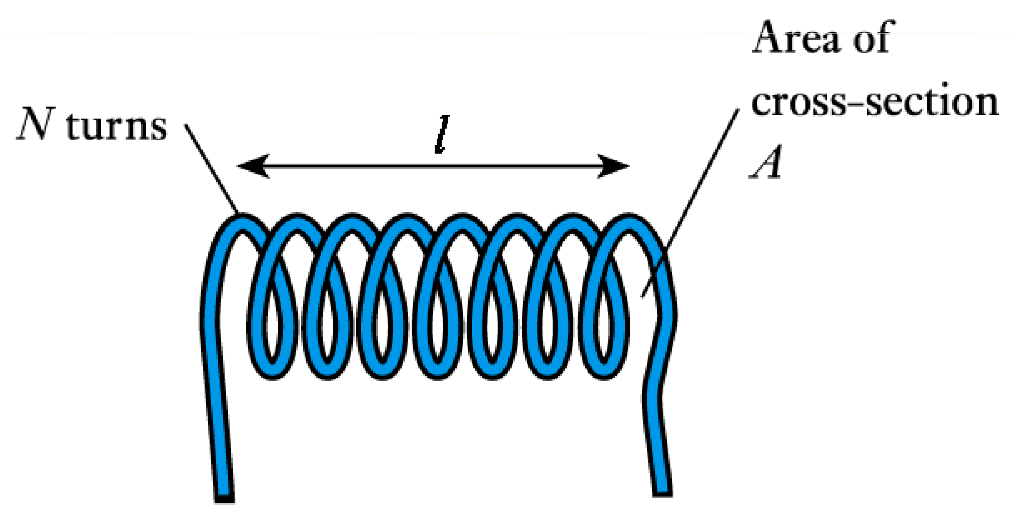
Potential difference across a resistor depends on the current

Potential difference across an inductor depends on the **rate of change** of the current

Inductor

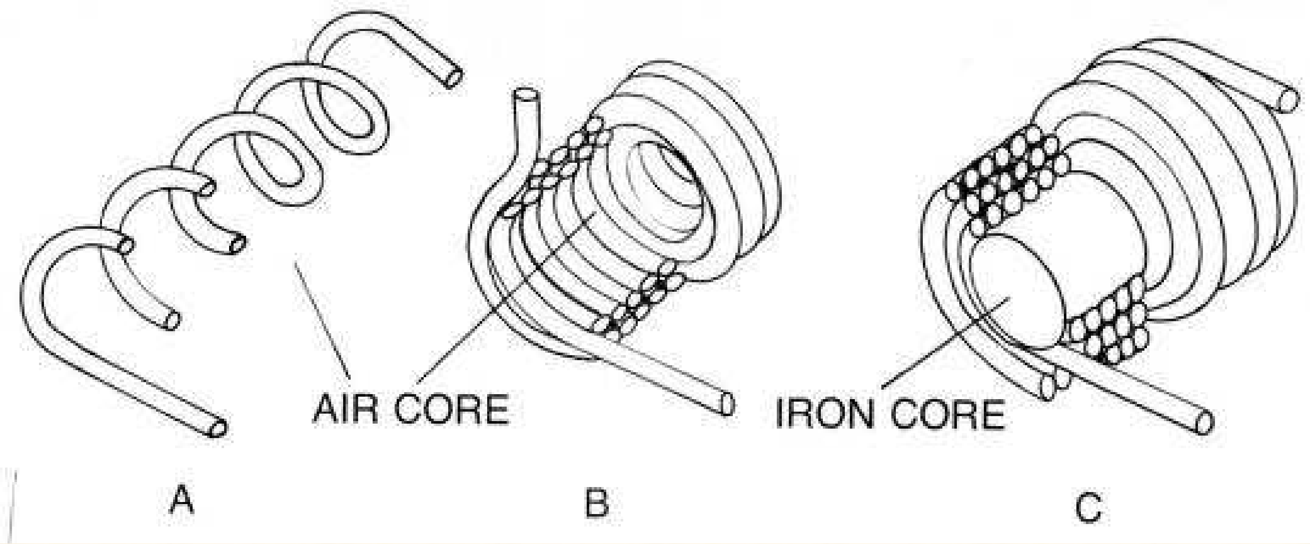
- The inductance of a coil depends on its dimensions and the materials around which it is formed

$$L = \frac{\mu_0 A N^2}{l}$$



(a) An air-filled coil

Types of Inductors



$$L = \frac{\mu_0 \mu_r A N^2}{l} = \frac{N^2}{\Re} = N^2 P$$

$$\Re = \frac{1}{P} = \frac{l}{\mu_0 \mu_r A}$$

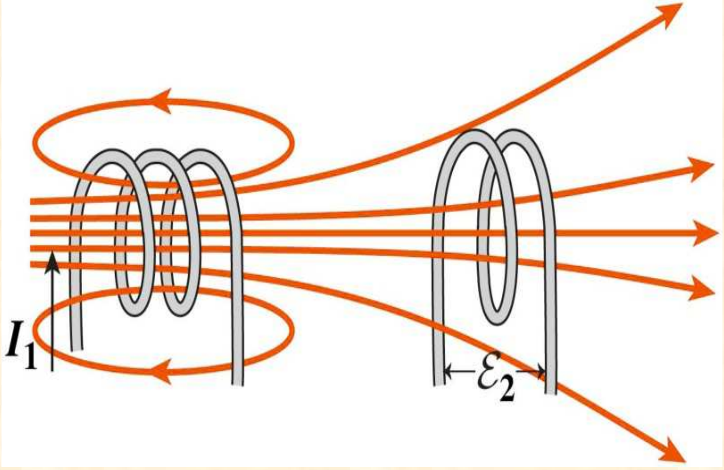
Factors Affecting Inductance of Coils

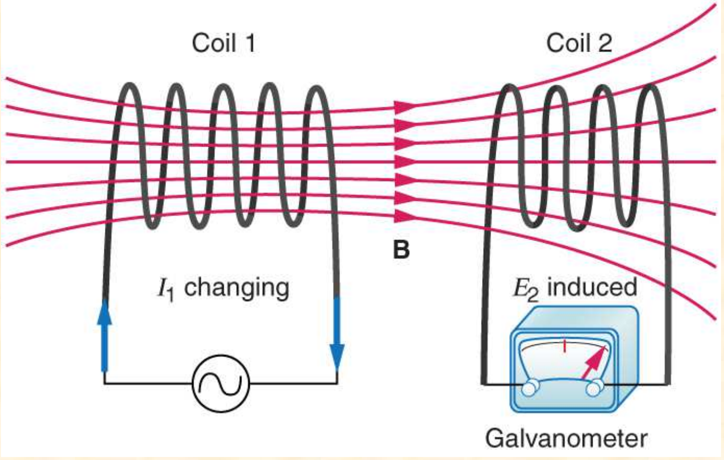
- **Numbers of Turns-** Inductance varies <u>directly</u> with the square of the number of turns
- **Permeability of Core-** Inductance varies <u>directly</u> with the permeability of the core
- **Cross-sectional Area of Core-** Inductance varies <u>directly</u> with the cross-sectional area of the core
- **Length of Core-** Inductance varies <u>inversely</u> with the length of the core

Mutual Inductance

- **Mutual inductance** occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit

  – Mutual inductance occurs because some of the magnetic flux produced by one circuit passes through the other circuit





Mutual Inductance

- When two inductors or coils are in close proximity to each other, magnetic flux caused by current in one coil links with the other coil, therefore producing the induced voltage

- The coils are said to have mutual inductance M, which can either add or subtract from the total inductance depending on if the fields are aiding or opposing

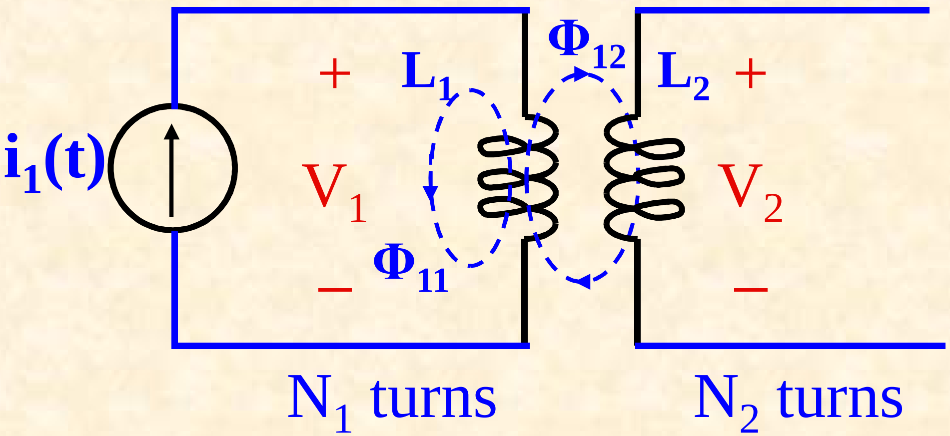
- **Mutual inductance** is the ability of one inductor to induce a voltage across a neighboring inductor

Mutual Inductance

Consider the following two cases:

- **Case 1:**

two coil with self – inductance L_1 and L_2 which are in close proximity which each other. Coil 1 has N_1 turns, while coil 2 has N_2 turns.



Mutual Inductance

- Magnetic flux Φ_1 from coil 1 has two components;
  - * Φ_{11} links only coil 1
  - * Φ_{12} links both coils
- ✓ Hence; $\Phi_1 = \Phi_{11} + \Phi_{12}$ ………. (6)
- ✓ Thus; the voltage induces in coil 1

$$v_1 = N_1 \frac{d\phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt} \quad \ldots\ldots(7)$$

Mutual Inductance

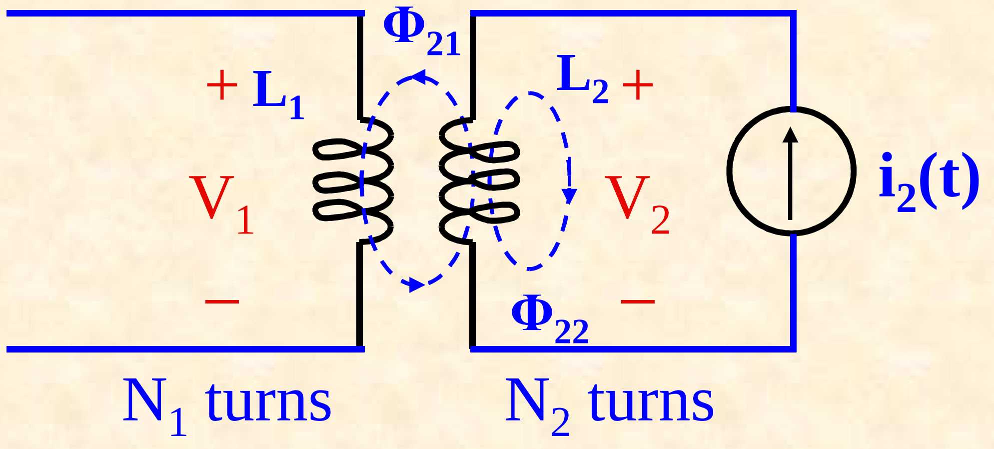
- ✓ The Voltage induces in coil 2

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt} \text{.......(8)}$$

Subscript 21 in M_{21} means the mutual inductance on coil 2 due to coil 1

Mutual Inductance

- **Case 2:** Same circuit but let current i_2 flow in coil 2.



✓ The magnetic flux Φ_2 from coil 2 has two components:

* Φ_{22} links only coil 2.

* Φ_{21} links both coils.

Hence; $\Phi_2 = \Phi_{21} + \Phi_{22}$ ……. (9)

Mutual Inductance

- ✓ Thus; the voltage induced in coil 2

$$v_2 = N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} = L_2 \frac{di_2}{dt} \text{.......(10)}$$

- ✓ the voltage induced in coil 1

$$v_1 = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt} \text{.......(11)}$$

Subscript 12 in M_{12} means the Mutual Inductance on coil 1 due to coil 2

Mutual Inductance

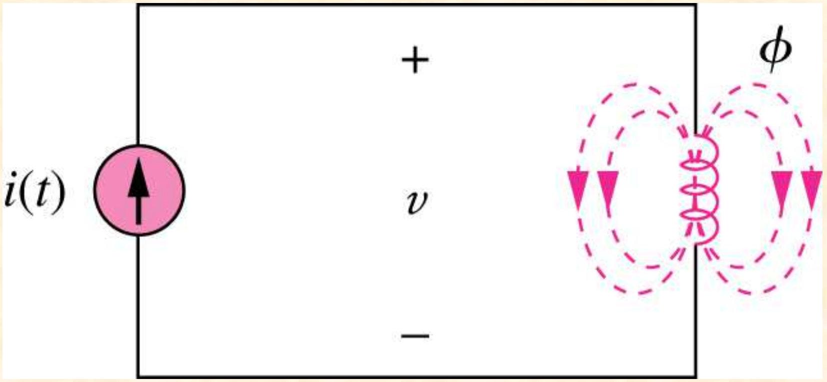
- Since the two circuits and two currents are the same:

For a linear system

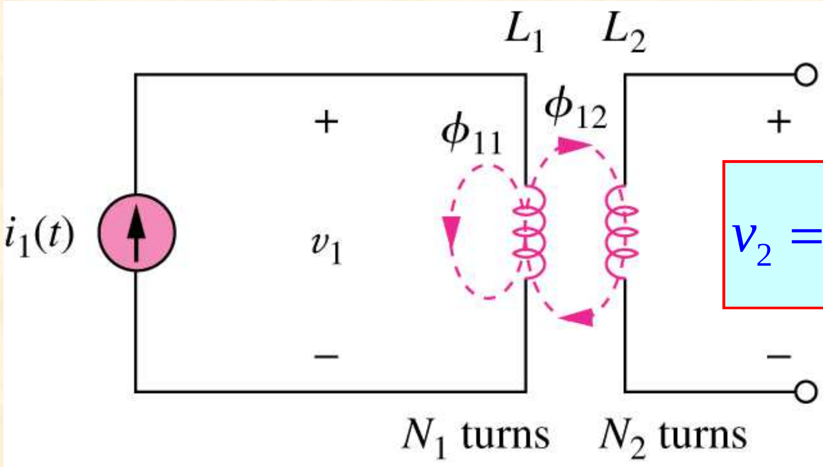
$$\boxed{M_{21} = M_{12} = M}$$

- Mutual inductance M is measured in Henrys (H)

Mutual Inductance (conclusions)



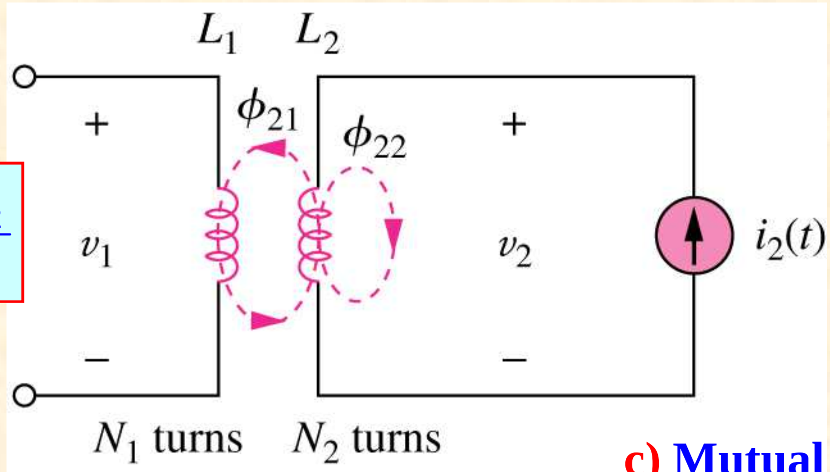
a) Magnetic flux produced by a single coil



$$\boxed{v_2 = M_{21}\frac{di_1}{dt}}$$

b) Mutual inductance M_{21} of coil 2 with respect to coil 1

$$\boxed{v_1 = M_{12}\frac{di_2}{dt}}$$



c) Mutual inductance of M_{12} of coil 1 with respect to coil 2

Terms & Definitions

- **Inductor-** A device that introduces inductance into an electrical circuit (usually a coil)
- **Inductance-** The property of an electric circuit when a varying current induces an EMF in that circuit or another circuit
- **Self-inductance-** The property of an electric circuit when an EMF is induced in that circuit by a change of current
- **Henry -** The unit of inductance
- **Permeability-** The measure of the ease with which material will pass lines of flux
- **Mutual Inductance-** The property of two circuits whereby an EMF is induced in one circuit by a change of current in the other